NASC 108 Name:______________________

The Rotation of Saturn and its Rings

When a light source is moving toward or away from an observer, the lines in its spectrum are displaced by an amount proportional to the speed of approach or recession. The famous red shift in the spectra of distant and receding galaxies is but one astronomical example of this so-called Doppler effect.

In this laboratory exercise, we investigate the Doppler effect in a spectrum of Saturn and its rings in order to deduce (1) the rotational periods, (2) the nature of the rings, and (3) the mass of the planet.

The accompanying spectrogram of Saturn and its rings, one of the finest in existence, provides the basis of our measurements. When it was taken on August 19, 1964, the rings were tilted to the line of sight from Earth by nine degrees. The spectrograph slit was oriented along the major axis of the rings; thus, it fell across the planet's disk and intercepted the rings on each side (see figure). The noticeable change in brightness in the outer parts of the rings marks the edge of the bright ring; Cassini's division between this and the outer ring becomes more evident when you sight along the length of the spectrogram nearly in the plane of the paper.

The inclined appearance of the dark spectrum lines is obvious at once, and its explanation rests on the Doppler principle. One edge of Saturn's disk is rotating away from the observer, and therefore light reflected from this edge has a shift toward the red or longer wavelengths of the spectrum. The other edge is rotating toward the observer and yields a blue shift.

Sharp-eyed students will note that not all the dark lines show a Doppler tilt, in particular a series near 628.0nm and at longer wavelengths. These are telluric lines, which originate in the Earth's own atmosphere. This particular series arises from molecular oxygen.

Each bright neon comparison line at the top of the picture has its counterpart at the bottom. Since these comparison lines are from lamps attached to the telescope, they are not affected by the Doppler shift and thus provide a standard for measurement of the inclination of the Saturnian spectral lines. To measure the inclination of the spectral lines, place the edge of a sheet of paper over the centers of the top and bottom comparison line having a wavelength of 621.728nm and secure with tape; this will be your reference line for your measurements.

Next, measure the distance from the reference line (the edge of the piece of paper) to the upper and lower edges of the line at 623.073nm in Saturn's disc (not those of the rings). Try to estimate to the nearest tenth of a millimeter.

$X1 =$__________ mm

$X2 =$__________ mm

Now take the difference between these two measurements.

$\Delta X = X1 - X2 =$____________ mm

This difference represents the tilt of the line. This measurement must now be converted to units of nano-meters.

In order to convert this difference from millimeters into wavelength units, it is necessary to establish the scale of the photograph in angstroms per millimeter by using the labeled neon comparison lines. The scale tells us how many nano-meters there are in the spectrum on each millimeter of the photograph. (This is similar to the scale on a road map which tells us how many miles on the ground correspond to each inch on the printed map.) Use the neon reference lines having wavelengths of 626.650 and 630.479nm and measure the separation of the two lines in millimeters.

Separation = ______________ mm

$Scale = \frac{630.479 - 626.650}{separation} =$_________________ nm/mm

The difference ΔX is converted into Δλ(nm) by multiplying the difference by the scale.

Δλ(nm) = ΔX(mm) x Scale = ________ x _________ = _________ Å

The velocity of approach or recession is then determined from the Doppler formula

$$V = \frac{\Delta\lambda}{\lambda} c \qquad \text{Equation 1}$$

where λ = rest wavelength of line being measured = 623.0.73nm.   c = speed of light = 300,000 kilometers per second.

Calculate the velocity of approach/recession.

$V = \frac{\quad}{623.073} \cdot 300{,}000 =$______________ km/s

Interpretation. In the case of a planet shining by reflected light, the displacement of the spectral lines is the sum of the rotation effects with reference to the Sun as the source and with reference to the observer. When the planet is at opposition (as is true here), the lines are shifted once because of motion with respect to the Sun and a second time because of motion with respect to the Earth. Thus, the displacement is doubled by reflection. Furthermore, since one limb of the planet is receding and the other approaching, the total displacement from one end of the spectrum to the other represents twice the equatorial velocity. Consequently, the measured velocity gives a value that is four times the equatorial velocity. Calculate the equatorial velocity.

Equatorial Velocity = ________/4 = ____________ km/s

Knowing the equatorial velocity and the radius of Saturn (60,268 kilometers) it is a simple matter to calculate the period of the planet's rotation in hours. With our comparatively simple measurements, however, the error may well be as great as an hour. The accepted equatorial period is 10.23 hours. The relationship between the equatorial velocity, radius, and period is

$$Period = 2 \cdot PI \cdot Radius / EquatorialVelocity$$

$$PI = 3.1416$$

Calculate the period in seconds.

$Period = 2 \cdot PI \cdot 60,268/$ ____________ = _____________ seconds

Convert the period from units of seconds to hours.

Period = ____________/3600 sec/hour = __________ hours

Calculate the percent error.

$$\%error = \frac{10.23 - (\_\_\_\_\_\_\_\_)}{10.23} \cdot 100 = $$ ______________ %

Saturn's Rings

If the rings rotated as a rigid structure, a given spectral line in the two ring spectra would fall on a single straight line. Clearly this is not the case, and some other hypothesis is required. James Keeler, writing in the first volume of The Astrophysical Journal (1895) described such an alternative:

"The hypothesis that the rings of Saturn are composed of an immense multitude of comparatively small bodies, revolving around Saturn in circular orbits, has been firmly established since the publication of Maxwell's classical paper in 1859. All the observed phenomena of the rings are naturally and completely explained by it, and mathematical investigation shows that a solid or fluid ring could not exist under the circumstances in which the actual ring is placed.

"I have recently obtained a spectroscopic proof of the meteoric constitution of the ring, which is of interest because it illustrates in a very beautiful manner the fruitfulness of Doppler's principle.

"Since the relative velocities of different parts of the ring would be essentially different under the two hypotheses of rigid structure and meteoric constitution, it is possible to distinguish between these hypotheses by measuring the motion of different parts of the ring in the line of sight."

The velocity V of a particle as a function of the distance r from a planet's center for particles moving according to Kepler's 3rd law is given by

$$V = \sqrt{\frac{GM}{r}} \qquad \text{Equation 2}$$

Here G, the constant of universal gravitation, is 6.67 x 10^{-20} in units of kilometers, kilograms, and seconds, while M is the mass of the planet. This equation is given as the dotted curve in the figure.

It is evident that the rings lie on the portion of the curve that is nearly a straight line. The inner particles revolve faster than the outer ones, so the spectral lines have the opposite tilt to the lines of the planet itself. Obviously, this is conclusive evidence that the ring system cannot be a single solid body, but must be composed of separately moving particles.

Proceeding as before, measure the displacements of the spectral lines in the rings. Begin with the inner edge of the ring by measuring the line at 6230.726 Å.

$X3 =$ __________ mm

$X4 =$ __________ mm

Calculate the displacement in millimeters.

$\Delta X = X3 - X4 =$ _______ - _______ = __________ mm

Convert the displacement to wavelength units.

$\Delta\lambda = \Delta X \cdot Scale =$__________ x ___________ = _____________ nm

Calculate the velocity of recession/approach using the Doppler formula.

$V = \frac{______}{623.073} \cdot 300{,}000 =$_____________ km/s

As before the velocity must be divided by 4.

V = ____________/4 = _____________ km/s

Calculate the period of rotation using the radius of the inner edge of the rings (74,000 km).

$Period = 2 \cdot PI \cdot 74{,}000/V =$______________ seconds

In units of hours.

Period = ___________/3600 = ____________ hours

Repeat the same process for the outer edge of the ring. The outer radius of the rings is 136,800 km.

$X5 =$_____________

$X6 =$_____________

$\Delta X =$_____________

$\Delta\lambda =$_____________

$V =$_____________

$V/4 =$_____________

$Period =$_____________ hours

Extra Credit Problem: Estimate the rotational period at the Cassini division from the above results for the inner and outer edges of the rings. How does this compare with the rotational period of Saturn's satellite, Mimas, which revolves about the planet inn 22.6 hours? The periods of Mimas and Cassini's division are said to be commensurable. Every second time around, a particle in the division finds Mimas in the same position. The resulting rhythmic gravitational attraction serves to drive any such particle into a new orbit, thereby leaving the gap between the rings.

Saturn's Mass

The mass of Saturn will be determined using the velocities calculated for the rings. Two values of the mass will be determined by applying the formula

$$Mass = Radius \times (Velocity)^2 / 6.67 \times 10^{-20}.$$

In symbol form

$$M = R \cdot V^2 / 6.67 \times 10^{-20}.$$

Using the velocity for the inner edge of the ring (R = 74,000 km) calculate the mass of Saturn.

$$M = 74{,}000 \cdot (\_\_\_\_\_\_\_\_\_\_\_\_)^2 / 6.67 \times 10^{-20} = \_\_\_\_\_\_\_\_\_\_\_\_ \text{ kg}$$

Repeat this process for the outer edge of the ring using the appropriate radius.

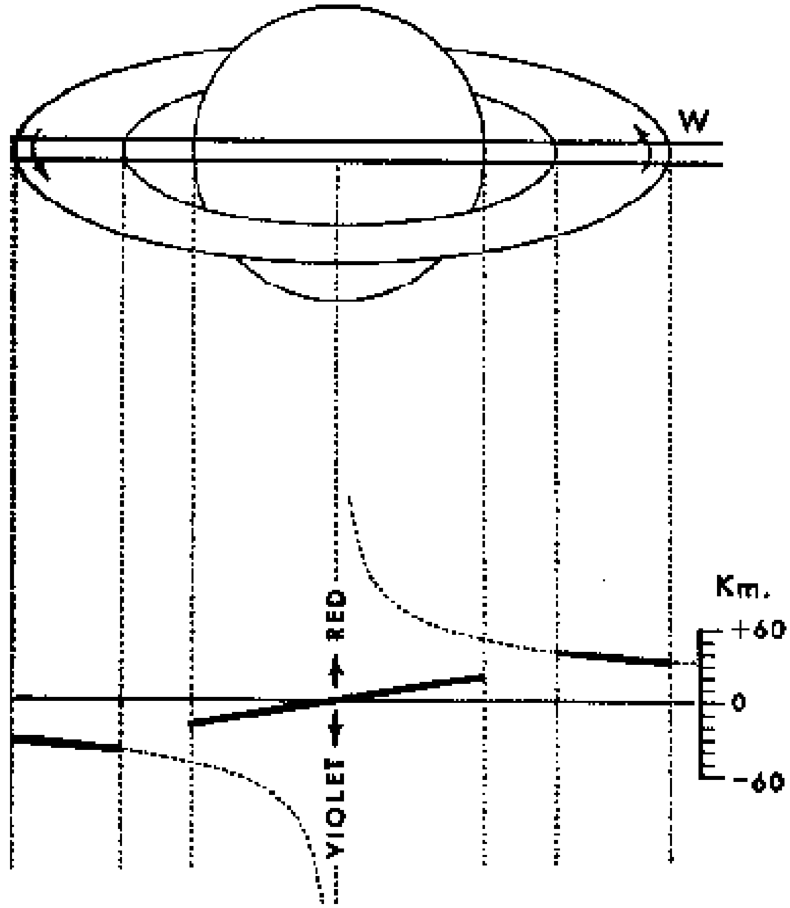
$$M = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Average the two values for the mass of Saturn (add them together and divide by two).

$$Ave\ \ Mass = \_\_\_\_\_\_\_\_\_\_\_\_\_$$

The accepted mass of Saturn is 5.7 x 10^{26}kg. Use this to find your percent error.

$$\%error = \frac{5.7 \times 10^{26} - (\_\_\_\_\_\_\_\_\_\_\_\_\_\_)}{5.7 \times 10^{26}} \cdot 100 = \_\_\_\_\_\_\_\_\_\_\_ \%$$


W
Km.
+60
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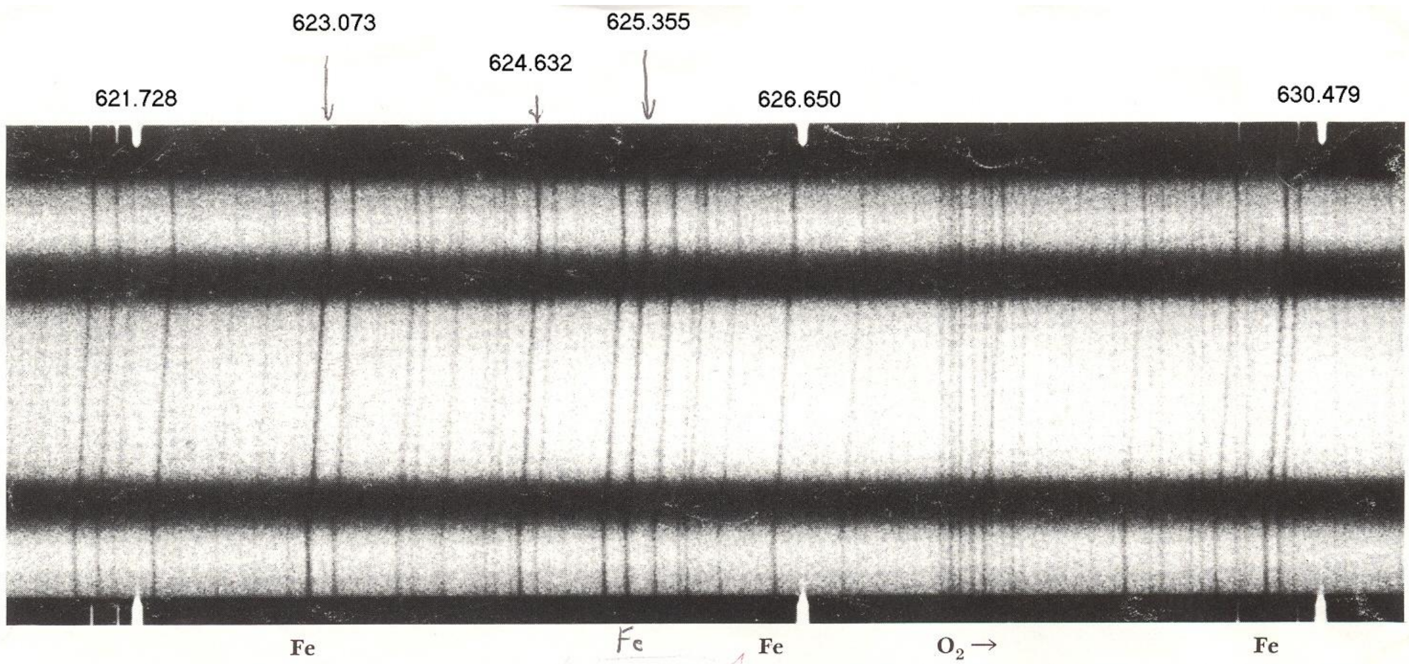

Letters at bottom identify elements causing particular absorption lines in sunlight reflected from Saturn's globe (broad central strip) and from its rings (narrower strips at top and bottom). The molecular oxygen lines originate not in the sun but in the earth's atmosphere after the light has been reflected by Saturn, so are not inclined. Lick Observatory photos.